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# ON AN INTENSITY-RATIO EQUIVALENCE FOR TWO TOP-QUARK DECAY COUPLINGS

Charles A. Nelson[1]

*Department of Physics, State University of New York at Binghamton*

*Binghamton, N.Y. 13902*

**Abstract**

For the $t \to W^+ b$ decay mode, an intensity-ratio equivalence for two distinct Lorentz-invariant couplings is shown to be a consequence of symmetries of tWb-transformations. Explicit tWb-transformations, $A_+ = M\ A_{SM}$, $P\ A_{SM}$, $B\ A_{SM}$ connect the four standard model's (SM) helicity amplitudes, $A_{SM}\left(\lambda_{W^+}, \lambda_b\right)$, and the amplitudes $A_+\left(\lambda_{W^+}, \lambda_b\right)$ in the case of an additional chiral-tensorial-coupling of relative strength $\Lambda_+ = E_W/2 \sim 53 GeV$. Such a coupling will arise if there is a large $t_R \to b_L$ chiral weak-transition-moment. Two commutator plus anti-commutator symmetry algebras are generated from $M, P, B$. Using these transformations, the associated mass scales are related to the SM's electroweak scale $v_{EW} \sim 246 GeV$.

[1]Electronic address: cnelson @ binghamton.edu

# 1 Introduction:

In this paper, for the $t \to W^+ b$ decay mode [1], an intensity-ratio equivalence [2] for two distinct Lorentz-invariant couplings is shown to be related to symmetries of tWb-transformations, $A_+ = M\ A_{SM}$, $P\ A_{SM}$, $B\ A_{SM}$, where $M, P, B$ are explicit 4x4 matrices. These tWb-transformations connect the standard model's helicity amplitudes, $A_{SM}(\lambda_{W^+}, \lambda_b)$, and the amplitudes $A_+(\lambda_{W^+}, \lambda_b)$ in the case of an additional chiral-tensorial-coupling of relative strength $\Lambda_+ = E_W/2 \sim 53GeV$. Versus the standard model's pure $(V-A)$ coupling, the additional $(f_M + f_E)$ tensorial coupling can be physically interpreted as arising due to a large $t_R \to b_L$ chiral weak-transition-moment for the t-quark. $\Lambda_+$ is defined by (1) below and the (+) amplitudes' complete coupling is (2). $E_W$ is the energy of the final W-boson in the decaying t-quark rest frame. The subscripts $R$ and $L$ respectively denote right and left chirality of the coupling, that is $(1 \pm \gamma_5)$. $\lambda_{W^+}$, $\lambda_b$ are the helicities of the the emitted W-boson and b-quark in the t-quark rest frame. The Jacob-Wick phase-convention [3] is used in specifying the phases of the helicity amplitudes and so of these transformations.

As tests with respect to the most general Lorentz coupling, in references [4,5], stage-two spin-correlation functions were derived and studied as a framework for complete measurements of the helicity parameters for $t \to W^+ b$ decay. Such tests are possible at the Tevatron [1], at the LHC [6], and at a NLC [7]. Due to rotational invariance, there are four independent $A(\lambda_{W^+}, \lambda_b)$ amplitudes for the most general Lorentz coupling. In this paper, a subset of the most general Lorentz coupling is considered in which the subscript "$i$" identifies the amplitude's associated coupling: "$i =$ SM" for the pure $(V-A)$ coupling, "$i = (f_M + f_E)$" for the pure $t_R \to b_L$ tensorial coupling, and "$i = (+)$" for $(V-A) + (f_M + f_E)$ with a t-quark chiral weak-transition moment of fixed relative

strength $\Lambda_+ = E_W/2$ versus $g_L$. Explicit expressions for the helicity amplitudes associated with each "$i$" coupling are listed in Sec. 2.

The Lorentz coupling involving both the SM's $(V-A)$ coupling and an additional $t_R \to b_L$ weak-moment coupling of arbitrary relative strength $\Lambda_+$ is $W_\mu^* J_{\bar{b}t}^\mu = W_\mu^* \bar{u}_b\,(p)\,\Gamma^\mu u_t\,(k)$ where $k_t = q_W + p_b$, and

$$\frac{1}{2}\Gamma^\mu = g_L\gamma^\mu P_L + \frac{g_{f_M+f_E}}{2\Lambda_+}\iota\sigma^{\mu\nu}(k-p)_\nu P_R \tag{1}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$. In $g_L = g_{f_M+f_E} = 1$ units, when $\Lambda_+ = E_W/2$ which corresponds to the $(+)$ amplitudes, the complete $t \to b$ coupling is very simple

$$\gamma^\mu P_L + \iota\sigma^{\mu\nu}\upsilon_\nu P_R = P_R\left(\gamma^\mu + \iota\sigma^{\mu\nu}\upsilon_\nu\right) \tag{2}$$

where $\upsilon_\nu$ is the W-boson's relativistic four-velocity.

We denote by $\Gamma$ the partial-width for the $t \to W^+b$ decay channel and by $\Gamma_{L,T}$ the partial-width's for the sub-channels in which the $W^+$ is respectively longitudinally, transversely polarized; $\Gamma = \Gamma_L + \Gamma_T$. Similarly, $\Gamma_L|_{\lambda_b=-\frac{1}{2}}$ denotes the partial-width for the W-longitudinal sub-channel with b-quark helicity $\lambda_b = -\frac{1}{2}$, etc.

The intensity-ratio equivalence statement is: "As consequence of Lorentz-invariance, for the $t \to W^+b$ decay channel each of the four ratios $\Gamma_L|_{\lambda_b=-\frac{1}{2}}/\Gamma$, $\Gamma_T|_{\lambda_b=-\frac{1}{2}}/\Gamma$, $\Gamma_L|_{\lambda_b=\frac{1}{2}}/\Gamma$, $\Gamma_T|_{\lambda_b=\frac{1}{2}}/\Gamma$, is identical for the pure $(V-A)$ coupling and for the $(V-A)+(f_M+f_E)$ coupling with $\Lambda_+ = E_W/2$, and their respective partial-widths are related by $\Gamma_+ = v^2\Gamma_{SM}$." $v \simeq 0.65$ is magnitude of the the velocity of the W-boson in the t-quark rest frame. The two couplings are obviously physically distinct because their associated amplitudes have different relative phases, see Table 1. This equivalence does not require specific values of the mass ratios $y \equiv m_W/m_t$, and $x \equiv m_b/m_t$, but $\Lambda_+ = E_W/2$ does determine the relative strength of the chiral weak-transition moment for the

t-quark versus $g_L$, as in (2) versus (1).

In the $t$ rest frame, the helicity-amplitude matrix element for $t \rightarrow W^+ b$ is $\langle \theta_1^t, \phi_1^t, \lambda_{W^+}, \lambda_b | \frac{1}{2}, \lambda_1 \rangle = D^{(1/2)*}_{\lambda_1,\mu}(\phi_1^t, \theta_1^t, 0) A_i(\lambda_{W^+}, \lambda_b)$ where $\mu = \lambda_{W^+} - \lambda_b$ in terms of the $W^+$ and $b$-quark helicities. The asterisk denotes complex conjugation, the final $W^+$ momentum is in the $\theta_1^t, \phi_1^t$ direction, and $\lambda_1$ gives the $t$-quark's spin component quantized along the $z$ axis. $\lambda_1$ is also the helicity of the $t$-quark if one has boosted, along the "$-z$" direction, back to the $t$ rest frame from the $(t\bar{t})_{cm}$ frame. It is this boost which defines the $z$ axis in the $t$-quark rest frame for angular analysis [4].

The three tWb-transformations, $A_+ = M\ A_{SM}$, $P\ A_{SM}$, $B\ A_{SM}$, are related to this equivalence statement. As explained in Sec. 2, the $M$ transformation implies the above equivalence statement, but postulating M also implies the sign and ratio differences of the (ii) and (iii) type amplitude ratio-relations which distinguish the (SM) and (+) couplings. The $P$ and $B$ transformations more completely exhibit the underlying symmetries relating these two Lorentz-invariant couplings. In particular, these three 4x4 matrices lead to two "commutator plus anti-commutator" symmetry algebras, and together can be used to relate the values of $\Lambda_+$, $m_t$, and $m_b$ to the SM's electroweak scale $v_{EW} = \sqrt{-\mu^2/|\lambda|} = \sqrt{2}\langle 0|\phi|0\rangle \sim 246 GeV$ where $\phi$ is the Higgs field.

In Sec. 2, it is shown how these three tWb-transformations successively arise from consideration of different types of "helicity amplitude relations" for $t \rightarrow W^+ b$ decay: The type (i) are ratio-relations which hold separately for the two cases, "$i = (SM), (+)$". The type (ii) are ratio-relations which relate the amplitudes in the two cases. From the type (iii) ratio-relations which also relate the two cases, the tWb-transformation $A_+ = M\ A_{SM}$ is introduced, where $M = v$

$diag(1,-1,-1,1)$ characterizes the mass scale $\Lambda_+ = E_W/2$. The amplitude condition (iv)

$$A_+(0,-1/2) = aA_{SM}(-1,-1/2), \tag{3}$$

with $a = 1 + O(v \neq y\sqrt{2}, x)$, determines the scale of the tWb-transformation matrix $P$ and determines the value of the mass ratio $y \equiv m_W/m_t$. $O(v \neq y\sqrt{2}, x)$ denotes small corrections, see below. The amplitude condition (v)

$$A_+(0,-1/2) = -bA_{SM}(1-1/2), \tag{4}$$

with $b = v^{-8}$, determines the scale of $B$ and determines the value of $x = m_b/m_t$. In Sec. 3, the two symmetry algebras are obtained which involve the $M$, $P$, and $B$ transformation matrices. Sec. 4 contains a discussion of these results and their implications assuming that the observed $t \to W^+b$ decay mode will be found empirically to be well-described by (2).

# 2 Helicity amplitude relations:

In the Jacob-Wick phase convention, the helicity amplitudes for the most general Lorentz coupling are given in [4]. In $g_L = g_{f_M+f_E} = 1$ units and suppressing a common overall factor of $\sqrt{m_t\,(E_b+q_W)}$, for only the $(V-A)$ coupling the associated helicity amplitudes are:

$$\begin{aligned}
A_{SM}\left(0,-\frac{1}{2}\right) &= \frac{1}{y}\,\frac{E_W+q_W}{m_t} \\
A_{SM}\left(-1,-\frac{1}{2}\right) &= \sqrt{2} \\
A_{SM}\left(0,\frac{1}{2}\right) &= -\frac{1}{y}\,\frac{E_W-q_W}{m_t}\left(\frac{m_b}{m_t-E_W+q_W}\right) \\
A_{SM}\left(1,\frac{1}{2}\right) &= -\sqrt{2}\left(\frac{m_b}{m_t-E_W+q_W}\right)
\end{aligned}$$

For only the $(f_M + f_E)$ coupling, i.e. only the additional $t_R \to b_L$ tensorial coupling:

$$
\begin{aligned}
A_{f_M+f_E}\left(0,-\frac{1}{2}\right) &= -(\frac{m_t}{2\Lambda_+})\; y \\
A_{f_M+f_E}\left(-1,-\frac{1}{2}\right) &= -(\frac{m_t}{2\Lambda_+})\sqrt{2}\;\frac{E_W+q_W}{m_t} \\
A_{f_M+f_E}\left(0,\frac{1}{2}\right) &= (\frac{m_t}{2\Lambda_+})y\left(\frac{m_b}{m_t-E_W+q_W}\right) \\
A_{f_M+f_E}\left(1,\frac{1}{2}\right) &= (\frac{m_t}{2\Lambda_+})\sqrt{2}\;\frac{E_W-q_W}{m_t}\left(\frac{m_b}{m_t-E_W+q_W}\right)
\end{aligned}
$$

From these, the amplitudes for the $(V-A)+(f_M+f_E)$ coupling of (1) are obtained by $A_+(\lambda_W,\lambda_b) = A_{SM}(\lambda_W,\lambda_b) + A_{f_M+f_E}(\lambda_W,\lambda_b)$. For $\Lambda_+ = E_W/2$, the $A_+(\lambda_W,\lambda_b)$ amplitudes corresponding to the complete $t \to b$ coupling (2) are

$$
\begin{aligned}
A_+\left(0,-\frac{1}{2}\right) &= \frac{1}{y}\,(q/E_W)\;\frac{E_W+q_W}{m_t} \\
A_+\left(-1,-\frac{1}{2}\right) &= -\sqrt{2}\,(q/E_W) \\
A_+\left(0,\frac{1}{2}\right) &= \frac{1}{y}\,(q/E_W)\;\frac{E_W-q_W}{m_t}\left(\frac{m_b}{m_t-E_W+q_W}\right) \\
A_+\left(1,\frac{1}{2}\right) &= -\sqrt{2}\,(q/E_W)\;\left(\frac{m_b}{m_t-E_W+q_W}\right)
\end{aligned}
$$

For each of the three "$i$" couplings, a direct derivation from (1) shows how the different factors arise in the amplitudes [8].

We now analyze the different types of helicity amplitude relations involving both the SM's amplitudes and those in the case of the $(V-A)+(f_M+f_E)$ coupling: The first type of ratio-relations holds separately for $i = (SM)$, $(+)$ and for all $y = \frac{m_W}{m_t}$, $x = \frac{m_b}{m_t}$, $\Lambda_+$ values, (i):

$$
\frac{A_i(0,1/2)}{A_i(-1,-1/2)} = \frac{1}{2}\frac{A_i(1,1/2)}{A_i(0,-1/2)} \tag{5}
$$

The second type of ratio-relations relates the amplitudes in the two cases and also holds for all $y, x, \Lambda_+$ values. The first two relations have numerators with opposite signs and denominators

with opposite signs, c.f. Table 1; (ii): Two sign-flip relations

$$\frac{A_+(0,1/2)}{A_+(-1,-1/2)} = \frac{A_{SM}(0,1/2)}{A_{SM}(-1,-1/2)} \tag{6}$$

$$\frac{A_+(0,1/2)}{A_+(-1,-1/2)} = \frac{1}{2}\frac{A_{SM}(1,1/2)}{A_{SM}(0,-1/2)} \tag{7}$$

and two non-sign-flip relations

$$\frac{A_+(1,1/2)}{A_+(0,-1/2)} = \frac{A_{SM}(1,1/2)}{A_{SM}(0,-1/2)} \tag{8}$$

$$\frac{A_+(1,1/2)}{A_+(0,-1/2)} = 2\frac{A_{SM}(0,1/2)}{A_{SM}(-1,-1/2)} \tag{9}$$

(7, 9), which are not in [2], are essential for obtaining the $P$ and $B$ tWb-transformations and thereby the symmetry algebras of Sec. 3 below.

The third type of ratio-relations, holding for all $y, x$ values, follows by determining the effective mass scale, $\Lambda_+$, so that there is an exact equality for the ratio of left-handed amplitudes (iii):

$$\frac{A_+(0,-1/2)}{A_+(-1,-1/2)} = -\frac{A_{SM}(0,-1/2)}{A_{SM}(-1,-1/2)}, \tag{10}$$

Equivalently, $\Lambda_+ = \frac{m_t}{4}[1 + (m_W/m_t)^2 - (m_b/m_t)^2] = E_W/2$ follows from each of:

$$\frac{A_+(0,-1/2)}{A_+(-1,-1/2)} = -\frac{1}{2}\frac{A_{SM}(1,1/2)}{A_{SM}(0,1/2)}, \tag{11}$$

$$\frac{A_+(0,1/2)}{A_+(1,1/2)} = -\frac{A_{SM}(0,1/2)}{A_{SM}(1,1/2)}, \tag{12}$$

$$\frac{A_+(0,1/2)}{A_+(1,1/2)} = -\frac{1}{2}\frac{A_{SM}(-1,-1/2)}{A_{SM}(0,-1/2)}, \tag{13}$$

From the amplitude expressions given above, the value of this scale $\Lambda_+$ can be characterized by postulating the existence of a tWb-transformation $A_+ = M\ A_{SM}$ where $M = v\ diag(1,-1,-1,1)$, with $A_{SM} = [A_{SM}(0,-1/2), A_{SM}(-1,-1/2), A_{SM}(0,1/2), A_{SM}(1,1/2)]$ and analogously for $A_+$.

Assuming (iii), the fourth type of relation is the equality (iv):

$$A_+(0,-1/2) = aA_{SM}(-1,-1/2), \tag{14}$$

where $a = 1 + O(v \neq y\sqrt{2}, x)$.

This is equivalent to the W-boson velocity formula $v = ay\sqrt{2}\left(\frac{1}{1-(E_b-q_W)/m_t}\right) = ay\sqrt{2}$ for $m_b = 0$. In [2], for $a = 1$ it was shown that (iv) leads to a cubic equation with the solution $y = \frac{m_W}{m_t} = 0.46006$ ($x = 0$). The present empirical value is $y = 0.461 \pm 0.014$, where the error is dominated by the 3% precision of $m_t$. In [2], for $a = 1$ it was also shown that (iv) leads to $\sqrt{2} = v\gamma(1+v) = v\sqrt{\frac{1+v}{1-v}}$ so $v = 0.6506\ldots$ without input of a specific value for $m_b$. However, by Lorentz invariance $v$ must depend on $m_b$. Accepting (iii), we interpret this lack of dependence on $m_b$ to mean that $a \neq 1$ and in the Appendix obtain the form of the $O(v \neq y\sqrt{2}, x)$ corrections in $a$ as required by Lorentz invariance. The small correction $O(v \neq y\sqrt{2}, x)$ depends on both $x \equiv m_b/m_t$ and the difference $v - y\sqrt{2}$.

Equivalently, by use of (i)-(iii) relations, (14) can be expressed postulating the existence of a second tWb-transformation $A_+ = P\ A_{SM}$ where

$$P \equiv v \begin{bmatrix} 0 & a/v & 0 & 0 \\ -v/a & 0 & 0 & 0 \\ 0 & 0 & 0 & -v/2a \\ 0 & 0 & 2a/v & 0 \end{bmatrix} \tag{15}$$

The value of the parameter $a$ of (iv) is not fixed by (15).

The above two tWb-transformations do not relate the $\lambda_b = -\frac{1}{2}$ amplitudes with the $\lambda_b = \frac{1}{2}$ amplitudes. From (i) thru (iv), in terms of a parameter $b$, the equality (v):

$$A_+(0,-1/2) = -bA_{SM}(1,1/2), \tag{16}$$

is equivalent to $A_+ = B\ A_{SM}$

$$B \equiv v \begin{bmatrix} 0 & 0 & 0 & -b/v \\ 0 & 0 & 2b/v & 0 \\ 0 & v/2b & 0 & 0 \\ -v/b & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

The choice of $b = v^{-8} = 31.152$, gives

$$B \equiv v \begin{bmatrix} 0 & 0 & 0 & -v^{-9} \\ 0 & 0 & 2v^{-9} & 0 \\ 0 & v^9/2 & 0 & 0 \\ -v^9 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

and corresponds to the mass relation $m_b = \frac{m_t}{b}\left[1 - \frac{vy}{\sqrt{2}}\right] = 4.407...GeV$ for $m_t = 174.3GeV$.

# 3 Commutator plus anti-commutator symmetry algebras:

The anti-commuting matrices

$$m \equiv \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, p \equiv \begin{bmatrix} 0 & -a/v \\ v/a & 0 \end{bmatrix}, q \equiv \begin{bmatrix} 0 & a/v \\ v/a & 0 \end{bmatrix} \tag{19}$$

satisfy $[m,p] = -2q, [m,q] = -2p, [p,q] = -2m$. Similarly, $m$ and

$$r \equiv \begin{bmatrix} 0 & -v/2a \\ 2a/v & 0 \end{bmatrix}, s \equiv \begin{bmatrix} 0 & v/2a \\ 2a/v & 0 \end{bmatrix} \tag{20}$$

are anti-commuting and satisfy $[m,r] = -2s, [m,s] = -2r, [r,s] = -2m$. Note $m^2 = q^2 = s^2 = 1$, $p^2 = r^2 = -1$, and that $a$ is arbitrary. Consequently, if one does not distinguish the (+) versus

SM indices, respectively of the rows and columns, the tWb-transformation matrices have some simple properties:

The anticommuting 4x4 matrices

$$M \equiv v \begin{bmatrix} m & 0 \\ 0 & -m \end{bmatrix}, P \equiv v \begin{bmatrix} -p & 0 \\ 0 & r \end{bmatrix}, Q \equiv v \begin{bmatrix} q & 0 \\ 0 & s \end{bmatrix} \tag{21}$$

satisfy the closed algebra $[\overline{M}, \overline{P}] = 2\overline{Q}, [\overline{M}, \overline{Q}] = 2\overline{P}, [\overline{P}, \overline{Q}] = 2\overline{M}$. The bar denotes removal of the overall "$v$" factor, $M = v\overline{M}, ....$ Note that $Q$ is not a tWb-transformation.

Including the B matrix with $b$ arbitrary, the algebra closes with 3 additional matrices

$$\overline{B} \equiv \begin{bmatrix} 0 & d \\ f & 0 \end{bmatrix}, \overline{C} \equiv \begin{bmatrix} 0 & e \\ g & 0 \end{bmatrix} \tag{22}$$

$$\overline{G} \equiv \begin{bmatrix} 0 & h \\ k & 0 \end{bmatrix}, \overline{H} \equiv \begin{bmatrix} 0 & j \\ l & 0 \end{bmatrix} \tag{23}$$

where

$$d \equiv \begin{bmatrix} 0 & -b/v \\ 2b/v & 0 \end{bmatrix}, e \equiv \begin{bmatrix} 0 & b/v \\ 2b/v & 0 \end{bmatrix}, f \equiv \begin{bmatrix} 0 & v/2b \\ -v/b & 0 \end{bmatrix}, g \equiv \begin{bmatrix} 0 & v/2b \\ v/b & 0 \end{bmatrix} \tag{24}$$

$$h \equiv \begin{bmatrix} -2ab/v^2 & 0 \\ 0 & b/a \end{bmatrix}, j \equiv \begin{bmatrix} 2ab/v^2 & 0 \\ 0 & b/a \end{bmatrix}, k \equiv \begin{bmatrix} 1/2v^2ab & 0 \\ 0 & -a/b \end{bmatrix}, l \equiv \begin{bmatrix} 1/2v^2ab & 0 \\ 0 & a/b \end{bmatrix} \tag{25}$$

The squares of the 2x2 matrices (24-25) do depend on $a$, $b$, and $v$.

The associated closed algebra is: $[\overline{M}, \overline{B}] = 0, \{\overline{M}, \overline{B}\} = -2\overline{C}; [\overline{B}, \overline{C}] = 0, \{\overline{B}, \overline{C}\} = -2\overline{M}$; $[\overline{M}, \overline{C}] = 0, \{\overline{M}, \overline{C}\} = -2\overline{B}$; and $[\overline{P}, \overline{B}] = 2\overline{H}, \{\overline{P}, \overline{B}\} = 0; [\overline{H}, \overline{P}] = 2\overline{B}, \{\overline{H}, \overline{P}\} = 0$;

$[\overline{H}, \overline{B}] = 2\overline{P}, \{\overline{H}, \overline{B}\} = 0$ . Similarly, $[\overline{P}, \overline{C}] = 0, \{\overline{P}, \overline{C}\} = -2\overline{G}; [\overline{M}, \overline{H}] = -2\overline{G}$, $\{\overline{M}, \overline{H}\} = 0; [\overline{H}, \overline{C}] = 0, \{\overline{H}, \overline{C}\} = 2\overline{Q}$; and $[\overline{M}, \overline{G}] = -2\overline{H}, \{\overline{M}, \overline{G}\} = 0; [\overline{P}, \overline{G}] = 0$, $\{\overline{P}, \overline{G}\} = 2\overline{C}; [\overline{G}, \overline{B}] = -2\overline{Q}, \{\overline{G}, \overline{B}\} = 0$; and $[\overline{G}, \overline{C}] = 0, \{\overline{G}, \overline{C}\} = -2\overline{P}$; $[\overline{G}, \overline{H}] = 2\overline{M}, \{\overline{G}, \overline{H}\} = 0$. The part involving $\overline{Q}$ is $[\overline{G}, \overline{Q}] = 2\overline{B}, \{\overline{G}, \overline{Q}\} = 0; [\overline{B}, \overline{Q}] = 2\overline{G}$, $\{\overline{B}, \overline{Q}\} = 0; [\overline{C}, \overline{Q}] = 0, \{\overline{C}, \overline{Q}\} = -2\overline{H}$; $[\overline{H}, \overline{Q}] = 0, \{\overline{H}, \overline{Q}\} = 2\overline{C}$.

This has generated an additional tWb-transformation $G \equiv v\overline{G}$; but $C \equiv v\overline{C}$ and $H \equiv v\overline{H}$ are not tWb-transformations.

Up to the insertion of an overall $\iota = \sqrt{-1}$, each of these 4x4 barred matrices is a resolution of unity, i.e. $\overline{P}^{-1} = -\overline{P}$, $\overline{G}^{-1} = -\overline{G}$, but $\overline{Q}^{-1} = \overline{Q}$, $\overline{B}^{-1} = \overline{B}$, ... .

# 4 Discussion:

**(1) $\Lambda_+$ mass scale:**

A fundamental question [2] raised by the existence of the intensity-ratio equivalence of two couplings for the $t \to W^+ b$ mode is "What is the origin of the $\Lambda_+ = E_W/2 \sim 53GeV$ mass scale?" The present paper shows that $M$ is but one of three logically-successive tWb-transformations which are constrained by the helicity amplitude ratio-relations (i) and (ii). Thereby, the type (iii) ratio-relation fixes $\Lambda_+ = E_W/2$ and the overall scale of the tWb-transformation matrix $M$. The amplitude condition (iv), $A_+(0, -1/2) = aA_{SM}(-1, -1/2)$ with $a = 1 + O(v \neq y\sqrt{2}, x)$, and the amplitude condition (v), $A_+(0, -1/2) = -bA_{SM}(1 - 1/2)$ with $b = v^{-8}$, determine respectively the scale of the tWb-transformation matrices $P$ and $B$ and characterize the values of $m_W/m_t$ and $m_b/m_t$. The overall scale can be set by choosing either $m_t$ or $m_W$. From an empirical "bottom-up"

perspective of further "unification", $m_W$ is more appropriate to use to set the scale since its value is fixed in the SM by the vacuum expectation value of the Higgs field, $\phi$.

From the $M$ transformation, the $\Lambda_+$ mass scale is fixed as $E_W/2$. From all three tWb-transformations, the numerical value of $\Lambda_+$ is determined by that of $v_{EW}$. Consequently like the value of the W-boson mass, the dimensional-analysis required $\Lambda_+$ scale of the $t_R \rightarrow b_L$ chiral weak transition-moment is not *in-itself* a "new mass scale" such as arising from a SUSY or a technicolor generalization of the SM, but instead the value of $\Lambda_+$ is another manifestation of the SM's electroweak scale.

**(2) Comparison with an Amplitude Equivalence-Theorem:**

Given the continued successes of predictions based on the couplings and symmetries built into the SM, and given the present rather slow pace of new experimental information, we appreciate the fact that for many readers it can be difficult to remind oneself that directly from experiment we still really do not know much about the properties of the on-shell t-quark [1]. Because of possible form factor effects and possible unknown thresholds due to new particles, in a theory/model-independent manner one cannot reliably determine the indirect constraints on on-shell t-quark couplings from off-shell contributions from t-quark contributions in higher-order loops in electroweak precision tests. Various assumptions of quark universality are still routinely made in the theoretical literature to conjecture on-shell t-quark properties from theoretical patterns found for the several-order-of-magnitude less massive quarks, in spite of the closeness of the value of $m_t$ to $v_{EW}$ and of the now significantly greater numerical-precision (3%) of the $m_t$ measurement than that of any of the other quark masses.

This present "not-knowing" status quo for the on-shell t-quark is very different from that in

1940-1952 in regard to the then rapidly changing and developing experimental status quo in the case of the weak and strong interactions, which was concurrent with the series of striking empirical-theoretical successes with QED ( QED is often viewed as the prototypical earlier analogue of the present SM ). Nevertheless, despite these differences in the experimental situation, we think it is instructive to compare this present intensity-ratio equivalence for two distinct t-quark decay couplings with a somewhat analogous "amplitude equivalence theorem (ET)" which was discovered and quite intensely studied, circa 1940-1952, for the pseudoscalar and pseudovector interactions, Dyson (1948) [9].

Although not as influential as Fermi's 1934 paper and the Gamow-Teller 1936 paper concerning the Lorentz structure of the weak interactions, this ET has had a long and significant impact in high energy experimental and theoretical physics. The early history of the ET , e.g. [9] and [10], can be traced from Schweber's book (1962) [11] and from the entire final chapter of Schweber, Bethe, and de Hoffmann, (1955) [12]. This ET stimulated in part, the development of effective Lagrangian methods [13] and work by G. t'Hooft and M. Veltman [14]. ( Later ET literature cites an t'Hooft-Veltman preprint; the corresponding published tHV-paper does not cite, e.g. [12], but refers to their tHV-paper as being based on unpublished preprints. ) The ET continues to be of theoretical interest, see for instance [15].

There are similarities and qualitative differences between this intensity-ratio equivalence of two t-quark couplings and the pseudoscalar-pseudovector ET. They are alike in that both relate simple Lorentz structures, involve helicities...though more intricately in the present case, and involve a renormalizable coupling. Major differences include (I) the ET case is much better understood after over 60 years of research, whereas this paper is only a beginning towards understanding

the symmetries and possible deeper physics implications/structure of this coupling-equivalence in t-quark decay, (II) the SM is now known to well explain most of the weak interaction systems (nucleon, nucleus, strange particle weak decays) first studied by the ET stimulating experiments, whereas precision experiments have only begun on t-quark decay, and (III) the tWb transformations involve couplings of a fundamental renormalizable local quantum field theory, the SM, and fundamental mass ratios, whereas, instead, the ET case involved hadronic couplings.

**(3) Possible Implications of These Symmetries:**

The presence in nature of an additional $t_R \rightarrow b_L$ weak-moment coupling would be but a further extension of the observed $(V - A)$ chiral asymmetry, i.e. a $(V + A) + (f_M - f_E)$ coupling versus a $(V + A)$ one would also lead to similar five types of analytical relations and a possible large $(f_M - f_E)$ chiral weak moment [8]. However, the additional $(f_M + f_E)$ coupling does violate the conventional gauge invariance transformations of the SM and in the past, in electroweak studies such anomalous couplings have been best considered as "induced" or "effective". Nevertheless, in special "new physics" circumstances such a simple charge-changing tensorial coupling as (2) might turn out to be a promising route to deeper understandings. Observation of a charge-changing "tensorial coupling" could prove to be a fundamental step forward from gravitation viewpoints.

An important formal, field-theoretic question is whether the new symmetries associated with the $(V - A) + (f_M + f_E)$ chiral structure such as the symmetry algebras of Sec. 3 are sufficient to overcome the known difficulties [16] in constructing a renormalizable, unitary quantum field theory involving second class currents [17] ? The $f_E$ component is second class. What might be of deeper significance in the context of CP violation, is that $f_E$ has a distinctively different reality structure, and time-reversal invariance property versus the first class $V, A, f_M$ components [18].

During 1970-1980 and later for tau-decay processes where $m_\tau \gg m_\nu$, various refinements in the classification of possible second class weak-interaction currents were suggested. Specific mechanisms/models were proposed for introducing such currents into the standard theory of the weak interactions [19]. Phenomenologically, the tWb-transformations can be viewed as another such attempt to generalize beyond the weak interaction couplings as embodied in the SM. If the observed $t \to W^+ b$ decay mode is found empirically to be well described by (2), this would then support a bolder working premise that in the on-shell limit of any treatment of t-quark decay and of $m_t$ and $m_b$, the underlying symmetries of these tWb-transformations are basic to relating the associated mass scales, much as are Lorentz invariance and the gauge-symmetries of $SU(2)XU(1)$ dynamics in performing calculations in the SM.

**(4) Experimental Tests/Measurements:**

For $t \to W^+ b$ decay channel there is a sizable $v^2$ factor difference of partial widths: the SM's $\Gamma_{SM} = 1.55 GeV$, versus $\Gamma_+ = 0.66 GeV$ and a longer-lived (+) t-quark if this mode is dominant. Complimentary to this on-shell process are the s-channel and t-channel off-shell processes in single-top-production at the Tevatron and LHC. If the SM description of t-quark phenomena is completely correct, measurement of CKM factor $|V_{tb}|^2$ will be possible via the s-channel process $qq' \to t\bar{b}$ in single-top production [20]. At present, from 162 $pb^{-1}$ of data, the CDF bound is 13.6pb versus the theoretical $0.88 \pm 0.11$pb (SM) prediction. From this, there is the estimate that the single-top-production process will be observed at the Tevatron when $1-2$ $fb^{-1}$ of data has been accumulated [1]. However, as is emphasized elsewhere in this paper, without assumptions concerning possible new particle thresholds and the off-shell behaviors of form factors, one can not reliably predict from $t \to W^+ b$ decay couplings what will occur in higher order, off-shell weak-

interaction precision tests nor in crossed channel reactions. In the literature, direct experimental tests for other than $(V-A)$ couplings in single-top-production have been proposed for both the s-channel and the t-channel (gluon-W fusion)[21]. In contrast to past searches for anomalous weak-interaction couplings for the other quarks and for the leptons, because the top mass is so large, the kinematics for all three of these processes (decay channel, single-top-production s-channel and t-channel) is excellent for direct searches for anomalous couplings. However, concurrently versus applications in light-quark and leptonic reactions, large and uncertain dispersion-theoretic extrapolations will be required in any attempt to relate measurements between these different t-quark reactions.

In on-going [1] and forth-coming [6,7] $t \to W^+ b$ experiments, important information about the relationship of the tWb-transformation symmetry patterns of this paper to the observed t-quark decay channel will come from:

(a) Tests for the existence of anomalous couplings and if found, for their structure and symmetries, such as from the on-going CDF/D0 analyses which investigate the W-boson helicity [1,4,22].

(b) Measurement of the sign of the $\eta_L \equiv \frac{1}{\Gamma}|A(-1,-\frac{1}{2})||A(0,-\frac{1}{2})| \cos\beta_L = \pm 0.46(SM/+)$ helicity parameter [4] so as to determine the sign of $cos\beta_L$ where $\beta_L = \phi^L_{-1} - \phi^L_0$ is the relative phase of the two $\lambda_b = -\frac{1}{2}$ amplitudes, $A(\lambda_{W^+}, \lambda_b) = |A| \exp(i\phi^{L,R}_{\lambda_{W^+}})$. For the exclusion of the coupling of (2) versus the SM's $(V-A)$ coupling, this would be the definitive near-term measurement concerning properties of the on-shell t-quark.

(c) Measurement, or an empirical bound, for the closely associated $\eta_L{}' \equiv \frac{1}{\Gamma}|A(-1,-\frac{1}{2})||A(0,-\frac{1}{2})| \sin\beta_L$ helicity parameter. This would provide useful complementary information, since in the absence of $T_{FS}$-violation, $\eta_L{}' = 0$ [4].

Since the helicity amplitude relations discussed in Sec. 2 involve the b-quark helicities, c.f. differing signs in $\lambda_b = 1/2$ column of Table 1, there are also independent phase tests which require (d) Measurements of helicity parameters [5] using $\Lambda_b$-polarimetry in stage-two spin-correlation functions. It is noteworthy that the $\Lambda_b$ baryon has been observed by CDF at the Tevatron [23].

For the case of spin-correlations in SM $t\bar{t}$ pair production and decay, several groups have investigated various higher-order corrections [24].

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**Appendix: The $O(v \neq y\sqrt{2}, x)$ corrections in $a$**

In this appendix is listed the form of the $O(v \neq y\sqrt{2}, x)$ corrections in $a$ as required by Lorentz invariance:

For $a = 1 + \varepsilon(x, y)$, the (iv) relation is $v = (1 + \varepsilon)y\sqrt{2}m_t/(E_W + q)$ whereas from relativistic kinematics $v = q/E_W = [(1 - y^2 - x^2)^2 - 4y^2x^2]^{1/2}/[1 + y^2 - x^2]$ . By equating these expressions and expanding in $x$, one obtains $\varepsilon = R + x^2 S$ where

$$
\begin{aligned}
R &= \frac{1 - 4y^2 - 3y^4 - 2y^6}{4y^2(1 + y^2)^2} \\
S &= \frac{-1 - 4y^2 + y^4}{2y^2(1 + y^2)^3}
\end{aligned}
$$

and $v = y\sqrt{2}\left[1 + R + x^2(S + \frac{1+R}{1-y^2}) + O(x^4)\right]$. From the latter equation, $R = (v - y\sqrt{2})/y\sqrt{2} + O(x^2)$.

For a massless b-quark ( $x = 0$ ) and $a = 1$ , the (iv) relation is equivalent to the $\frac{m_W}{m_t}$ mass relation $y^3\sqrt{2} + y^2 + y\sqrt{2} - 1 = 0$, and by relativistic kinematics to the W-boson velocity condition

$v^3 + v^2 + 2v - 2 = 0$ and the simple formula $v = y\sqrt{2}$.

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## Table Captions

Table 1: Numerical values of the helicity amplitudes for the standard model $(V-A)$ coupling and for the $(+)$ coupling of Eq.(2) which has a $(V-A)+(f_M+f_E)$ Lorentz-structure. The latter coupling consists of an additional $t_R \to b_L$ chiral weak transition-moment of relative strength $\Lambda_+ \sim 53GeV$ so as to produce a relative-sign change in the $\lambda_b = -\frac{1}{2}$ amplitudes. The values are listed first in $g_L = g_{f_M+f_E} = 1$ units, and second as $A_{new} = A_{g_L=1}/\sqrt{\Gamma}$. Table entries are for $m_t = 175GeV,\ m_W = 80.35GeV,\ m_b = 4.5GeV$.

Table 1: Helicity amplitudes for $(V - A)$ coupling and the $(+)$ coupling of Eq.(2).

| | $A(0,-\frac{1}{2})$ | $A(-1,-\frac{1}{2})$ | $A(0,\frac{1}{2})$ | $A(1,\frac{1}{2})$ |
|---|---|---|---|---|
| $\mathrm{A}_{g_L=1}$ in $g_L = 1$ units : | | | | |
| $(V - A)$ | 338 | 220 | $-2.33$ | $-7.16$ |
| $(V - A) + (f_M + f_E)$ | 220 | $-143$ | 1.52 | $-4.67$ |
| $\mathrm{A}_{New} = A_{g_L=1}/\sqrt{\Gamma}$ : | | | | |
| $(V - A)$ | 0.84 | 0.54 | $-0.0058$ | $-0.018$ |
| $(V - A) + (f_M + f_E)$ | 0.84 | $-0.54$ | 0.0058 | $-0.018$ |